A Multi-mode, Multi-class and Multi-criteria Dynamic Network Model with Queue for Advanced Transportation Information Systems

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Abstract: In this paper we propose a composite Variational Inequality formulation for modeling multi-mode, multi-class and multi-criteria stochastic dynamic user equilibrium problem in recurrent congestion network with queues. The each mode of travelers is classified into two classes. One is the equipped traveler following stochastic dynamic user-equilibrium with less uncertainty of travel cost, another is the unequipped traveler following stochastic dynamic user-equilibrium with more uncertainty of travel cost. We consider the each mode of both equipped and unequipped travelers are multi-criteria decision-maker in that they perceive their disutility associated with selecting routes as a weighting of the travel times and travel costs. A solution algorithm based on stochastic dynamic network loading for logit-based simultaneous route and departure time choice is proposed. Finally a numerical example is presented in a simple network.

Key Words: ATIS; Multi-mode Dynamic Network Model; Dynamic Traffic Assignment; Queuing Network

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1 Introduction

As a main part of intelligent transportation systems (ITS), Advanced traveler information systems that have experienced a rapid development in the past decade are generally believed to be a efficient means for improving individual traveler’s trip planning, alleviating traffic congestion and enhancing traffic network performance. There are many researches for modeling and evaluating the effect of ATIS on traveler and transportation system in order to determine the feasibility, risks and benefits of such technology. On the other hand, whether can the travelers who receive traffic information versus travelers who can’t accept traffic information capture the relevant travel cost savings or not? Whether can ATIS reduce travel times and congestion delays, traffic pollution that recently induce more and more attention or not? These researches mainly include field experiment (Tsuji et al 1985), traffic simulation (Emmerink 1995, J. Barcelo 1999, Mahmassani 1994, Q.chen 1997, Reddy.P 1995, U.Vandebona 1997, Wunderlich.K 1998), and analytical models and so on.

Analytical multi-class traffic models for evaluating the effects of ATIS on general network with recurrent congestion are classified into two classes. One class is static model (see, for example, Harker, 1988; Kanafani 1991, Van Vuren., 1991; Bennett, 1993; Maher and Hughes, 1995; and Yang, 1998 1999, Hong K.Lo 2002a) based on static traffic assignment theory. The static model is a rather simplistic model that may give unrealistic result for determining the benefits of ATIS due to its well-known flow instantaneous propagation property and do not permit traveler’s departure time, dynamic queuing locations and duration. The other class is dynamic models that mainly are based on dynamic traffic assignment theory. As compared with static models, dynamic models can reflects the problems, at the expense of significant increases in model complexity, computing time and. Hong K.Lo(2002b) compare the cell-based dynamic models and static counterpart in a simple network and indicate two classes of model can bring out the reverse result in similar cases.

Al-Deek, et al (1998) uses a composite traffic assignment model which combines a probabilistic traveler behavior model of route diversion and a queuing model to evaluate ATIS impacts under incident conditions. The composite assignment model considers three types of travelers: those who are unequipped with electronic devices, i.e. they do not have ATIS or radio in their vehicles; those who receive delay information from radio only; and those who access ATIS only. Similar studies are carried out by Emmerink.K (1994 1995a) in studying the economic impacts of driver information systems by using a dynamic stochastic route choice-modeling framework.

Among the first to use an analytical approach to modeling multi-class DTA were Lo et al. (1996) and Ran et al. (1996). In these papers, travelers were classified into those who follow predetermined routes, those who follow a stochastic dynamic UE assignment, and those who follow a dynamic UE assignment. The three classes of users are integrated into one dynamic traffic assignment (DTA) model Ran et al. (1996, 2002) gives various algorithms for solving the above multi-class equilibrium problem. Lam and Huang (2003) presents a multi-class dynamic user equilibrium assignment problem formulation in order to assess the impacts of
ATIS in general network with queues. Suppose that users equipped with ATIS will follow the deterministic simultaneous route/departure time equilibrium choice behavior due to complete traffic information, while users unequipped with ATIS will follow the stochastic simultaneous route/departure time equilibrium choice behavior (travel choice behavior modeling by the nested-logit model) due to incomplete traffic information. A heuristic algorithm based on route/time swapping process is processed for solving the multi-class dynamic user equilibrium problem. However, the previous analytical static or dynamic multi-class model did not consider the multiple transportation modes and multi-mode dynamic queuing phenomenon in general networks with ATIS.

Bliemer and Bovy (2000, 2001, 2003) firstly extend single mode dynamic traffic assignment to multi-mode dynamic assignment model, considering different driving characteristic, network usage and route choice behavior and give various solution algorithms such as extended time-space network algorithm, nested modified projection method and so on. In this paper we aim to propose a multi-mode dynamic network model for the case of multiple user classes in a queuing network with ATIS. We classify each mode of travelers into two classes; each mode of travelers equipped with ATIS devices who can receive the fairly precise traffic information will follow stochastic dynamic simultaneous route/departure time equilibrium choice behavior with less uncertainty of travel cost. However each mode of travelers equipped with ATIS devices who can capture only the imperfect traffic information (perhaps from past experiences) will follow stochastic dynamic simultaneous route/departure time equilibrium choice behavior with more uncertainty of travel cost. Here we propose stochastic dynamic equilibrium condition is a particular case of the nested-logited stochastic dynamic simultaneous route/departure time equilibrium condition given by Williams and Huang (2003).

We introduce multi-criteria travel choice in order to reflect travel route and departure time choice behaviors really. The use of multiple criteria in decision-making as regards making travel choices, and, in particular, route and departure time choice, allows one to not only capture travel time as a criteria, but also cost in the sense of the level of safety, comfort, pollutions and actual expense (fuel consumption). Multi-criteria traffic network models were proposed by Nagurney.A(2000,2001) and explicitly consider that travelers may be faced with several criteria, notably, travel time and travel cost, in selecting their optimal routes of travel. The remainder of this paper is organized as follows. In section 2, multimodal, multi-class and multi-criteria dynamic traffic network equilibrium model is developed on a discrete-time basis. In section 3, the governing multimode, multi-class and multi-criteria dynamic traffic network equilibrium conditions are formulated as finite-dimensional variational inequality problem. In section 4, we give a diagonalisation algorithm. Finally the model and algorithms are tested in a simple network, and investigate the ATIS impact on travelers and system performance. It should be noted that the model proposed in this paper is mainly used for planning and evaluation purposes particularly for assessing the impacts of ATIS in general networks with queues. Therefore, some of the assumptions adopted in the model may be appropriate under certain circumstances such as under recurrent congestion conditions without spillback effects at intersections. This may be true for commuter trips during normal peak hour periods.
2 Discrete-Time Network Model

Consider a network $G = (N; A)$, where $N$ is the set of nodes and $A$ is the set of links in the network. Let $a$ denote a link of the network connecting a pair of nodes $(i,j)$ and let $p$ denote a path that is consisted of a series of directed link $(a_1,a_2,\ldots,a_n)$ between origin $r$ and destination $s$. Let $RS$ denote the set of all OD pairs in the network. $P_{rs}$ denote the set of routes between OD pair $rs \in RS$ and the entire set of paths in the network by $P$. Let $M$ denote the set of all modes, examples of modes include passenger cars, trucks and public transportations, etc. The studies horizon is discretized into $m$ intervals of length $\delta$ such that $T = m \cdot \delta$. Here, we assume the study horizon is enough long to ensure all traveler can exit from the network after the time $T$. on the other hand, it is also assumed that the value of $\delta$ is small enough so that the discrete-time model can approximate its continuous time counterpart.

$\hat{u}_{an}(k), \hat{u}'_{an}(k)$ : the inflow rate of equipped and unequipped travelers of mode $m$ on link $a$ during time interval $k$

$\hat{a}_{an}(k), \hat{a}'_{an}(k)$ : the arrival flow rate to exit queue of equipped and unequipped travelers of mode $m$ on link $a$ during time interval $k$

$\hat{v}_{an}(k), \hat{v}'_{an}(k)$ : the departure flow rate of equipped and unequipped travelers of mode $m$ from link $a$ during time interval $k$

$\hat{q}_{an}(k), \hat{q}'_{an}(k)$ : the queue vehicle numbers of the travelers of mode $m$ on link $a$ at time interval $k$

$t_{am}(k)$ : the travel time experienced by the travelers of mode $m$ entering into link $a$ at time interval $k$

$s_a$ : the maximum exit flow rate of the bottleneck on link $a$ (unit: passenger car number of hour )

$Pcu_m$: the passenger car equivalents parameter of mode $m$ (here, denote $Pcu$ as the unit of passenger car)

2.1 Link Dynamic Model

We extend single-mode deterministic point queue model proposed by Kuwahara and Akamatsu(1993,1997), J.Li(2000), Huang(2002), S.Han(2003) to multi-mode deterministic point queue model. In other words, we don’t consider the spillback effect of queue length explicitly, the reader can be referred to the work of Astarita(1996), Adamo(1999), Kuwahara(2001), Hong.K(2002c) et al for handling the spillback of congestion in a dynamic network simulation model. We assume the link is consisted of two parts. The first part is the running segment of the link that the each mode of travelers can run according to the each mode of free-flow velocity and don’t interact with each other among the travelers of various modes. In other words, the each mode of travelers experiences the constant running time $t_{am}$ to the exit queue segment of link (there assume $t_{a1}<t_{a2}<\ldots<t_{aw}$, due to difference of the velocity of various modes).
modes in un-congested traffic condition. For example, the velocity of car is more than the velocity of truck in un-congested traffic condition). The second part is the exit queue segment (the vehicles of mode $m$ is assumed to be a point without length). The queue delay experienced by the travelers of mode $m$ is caused by the limited link exit capacity (in $Pcu$), or the maximum link exit flow rate (in $Pcu/h$).

The link flow propagation condition is depicted in Fig.1. The equipped and unequipped travelers of mode $m$ entering into link $a$ during time interval $k-t_{am}$ experience the constant running time $t_{am}$ to arrive at the exit queue segment of link $a$ during time interval $k$. On the other hand, the arrival flow rate of equipped and unequipped travelers of mode $m$ to the exit queue segment of link $a$ during time interval $k$ is $\hat{v}_{am}(k), \hat{v}_{am}(k)$. The departure flow rate of equipped and unequipped travelers of mode $m$ from the exit queue segment during time interval $k$ is $\hat{v}_{am}(t), \hat{v}_{am}(t)$. The link dynamic model is expressed as follows.

The running segment model:

$$\hat{v}_{am}(k) = \bar{u}_{am}(k-t_{am}), \forall a, k, m$$  \hspace{0.5cm} (1)  

$$\hat{v}_{am}(k) = \hat{u}_{am}(k-t_{am}), \forall a, k, m$$  \hspace{0.5cm} (2)

The exit queue segment model:

$$\frac{\hat{q}_{am}(k) - \hat{q}_{am}(k-1)}{\delta} = \hat{v}_{am}(k) - \hat{v}_{am}(k), \forall a, k, m$$  \hspace{0.5cm} (3)

$$\frac{\tilde{q}_{am}(k) - \tilde{q}_{am}(k-1)}{\delta} = \tilde{v}_{am}(k) - \tilde{v}_{am}(k), \forall a, k, m$$  \hspace{0.5cm} (4)

Let $\hat{q}_{am}(k), \tilde{q}_{am}(k)$ be the number of equipped and unequipped travelers of mode $m$ waiting in the queue on link $a$ at time interval $k$. In Equation (3) and (4), it is important to note that, for each class, the equipped travelers of mode $m$ for example, express the marginal change of queue length of equipped travelers of mode $m$ is equal to difference between the arrival flow rate of equipped travelers of mode $m$ to exit queue segment and the departure flow rate of equipped travelers of mode $m$ from exit queue segment on link $a$ during time interval $k$. 

2.2 Link Exit Model

We will use the following assumption for deriving our link exit model

(1). The class-specific $Pcu_m$ parameter that transforms the effect of mode $m$ into passenger car equivalents is fixed under all traffic condition.

(2). The mixture of the equipped and unequipped travelers of various modes is homogenous on the link.

The temporal and spatial interactions of the equipped and unequipped travelers of various modes mainly appear in the exit queue segment of the link. If the total queue length $q_a(k)$ ( in $Pcu$ , let $q_a(k) = \sum_m pcu_m (\dot{q}_{am}(k) + \ddot{q}_{am}(k))$ ) is equal to zero (the queue length of equipped and unequipped travelers of mode $m$ for all time interval is nonnegative, $\dot{q}_{am}(k) \geq 0, \ddot{q}_{am}(k) \geq 0, \forall a,m,k$ ), the queue length of equipped and unequipped travelers of mode $m$ at time interval $k$ is equal to zero, $\dot{q}_{am}(k) = 0, \ddot{q}_{am}(k) = 0, \forall a,m,k$. Thus the following equation can be got according to equation (3) and (4).

$$\dot{v}_{am}(k) = \dot{v}^*_m(k) + \frac{\dot{q}_{am}(k-1)}{\delta}, \forall a,k,m$$

(5)

$$\ddot{v}_{am}(k) = \ddot{v}^*_m(k) + \frac{\ddot{q}_{am}(k-1)}{\delta}, \forall a,k,m$$

(6)

On the other hand, If the total queue length $q_a(k)$ ( in $Pcu$ ) is more than zero; In other words, due to the limited exit capacity (in $Pcu/h$), total flow that hope to exit the link during time interval $k$, $v_a(k)$ ( in $Pcu$ ) ,

let $v_a(k) = \sum_m pcu_m (\dot{v}_{am}(k) + \dot{v}_{am}(k)) + \sum_m pcu_m (\dot{v}_{am}(k-1) + \dot{v}_{am}(k-1))/\delta$ is more than maximum exit flow, $v_a(k) > s_a$ , some part of flows will only be allowed to exit link and other flow will form the new queue at the exit queue segment of link. According to the above assumption and equation (3) and (4), The departure flow rate of equipped and unequipped travelers of mode $m$ are calculated as follows.

$$\dot{v}_{am}(k) = \begin{cases} \dot{v}_{am}(k) + \dot{q}_{am}(k-1)/\delta \cdot s_a & \text{if } v_a(k) \geq s_a \text{ or } q_a(k) = 0 \\
\dot{v}_{am}(k) + \dot{q}_{am}(k-1)/\delta & \text{otherwise} \end{cases} \forall a,k,m$$

(7)

$$\ddot{v}_{am}(k) = \begin{cases} \ddot{v}_{am}(k) + \ddot{q}_{am}(k-1)/\delta \cdot s_a & \text{if } v_a(k) \geq s_a \text{ or } q_a(k) = 0 \\
\ddot{v}_{am}(k) + \ddot{q}_{am}(k-1)/\delta & \text{otherwise} \end{cases} \forall a,k,m$$

(8)

The link queue length of equipped and unequipped travelers of mode $m$ is given as
\[ \hat{q}_{am}(k) = \begin{cases} \hat{v}_{am}(k) \cdot \delta + \hat{q}_{am}(k-1) - \frac{\hat{v}_{am}(k) \cdot \delta + \hat{q}_{am}(k-1)}{v_a(k)} \cdot s_a & \text{if } v_a(k) \geq s_a \text{ or } q_a(k) = 0 \quad \forall a, k, m \quad (9) \\ 0 & \text{otherwise} \end{cases} \]

\[ \tilde{q}_{am}(k) = \begin{cases} \hat{v}_{am}(k) \cdot \delta + \tilde{q}_{am}(k-1) - \frac{\hat{v}_{am}(k) \cdot \delta + \tilde{q}_{am}(k-1)}{v_a(k)} \cdot s_a & \text{if } v_a(k) \geq s_a \text{ or } q_a(k) = 0 \quad \forall a, k, m \quad (10) \\ 0 & \text{otherwise} \end{cases} \]

Equation (9), (10), (11), (12) can be expressed as follows too.

\[ \hat{v}_{am}(k) = \min \left\{ \hat{v}_{am}(k) + \hat{q}_{am}(k-1) - \frac{\hat{v}_{am}(k) + \hat{q}_{am}(k-1)}{v_a(k)} \cdot s_a \right\}, \forall a, k, m \quad (11) \]

\[ \tilde{v}_{am}(k) = \min \left\{ \hat{v}_{am}(k) + \tilde{q}_{am}(k-1) - \frac{\hat{v}_{am}(k) + \tilde{q}_{am}(k-1)}{v_a(k)} \cdot s_a \right\}, \forall a, k, m \quad (12) \]

\[ \hat{q}_{am}(k) = \max \left\{ 0, \hat{v}_{am}(k) \cdot \delta + \hat{q}_{am}(k-1) - \frac{\hat{v}_{am}(k) \cdot \delta + \hat{q}_{am}(k-1)}{v_a(k)} \cdot s_a \right\}, \forall a, k, m \quad (13) \]

\[ \tilde{q}_{am}(k) = \max \left\{ 0, \tilde{v}_{am}(k) \cdot \delta + \tilde{q}_{am}(k-1) - \frac{\tilde{v}_{am}(k) \cdot \delta + \tilde{q}_{am}(k-1)}{v_a(k)} \cdot s_a \right\}, \forall a, k, m \quad (14) \]

Some definitional constraints

\[ v_{am}(k) = \hat{v}_{am}(k) + \tilde{v}_{am}(k) \quad \forall a, k, m \quad (15) \]

\[ \hat{v}_{am}(k) = \hat{v}_{am}(k) + \tilde{v}_{am}(k) \quad \forall a, k, m \quad (16) \]

\[ q_{am}(k) = \hat{q}_{am}(k) + \tilde{q}_{am}(k) \quad \forall a, k, m \quad (17) \]

### 2.3 Link Travel Time Model

The queue delay experienced by the travelers of mode \( m \) is depended on the link total queue length, \( q_a(k) \) (in Pcu). The travelers of mode \( m \) entering into link \( a \) during time interval \( k \) will spend a constant running time \( t_{am} \) on the link. They will reach the exit queue segment of link \( a \) during time interval \( k + t_{am} \) and the total queue length at time interval \( k + t_{am} \) is \( q_a(k + t_{am}) \).

Hence, The link delay experienced by the travelers of mode \( m \) entering into link \( a \) during time interval \( k \) is given as
The total travel time over link $a$ for the travelers of mode $m$ entering into link $a$ during time interval $k$ is the sum of constant running time and the delay experienced by the travelers of mode $m$

$$t_{am}(k) = t_{am} + d_{am}(k) = t_{am} + \frac{q_a(k+t_{am})}{s_a} \quad \forall a, k, m \tag{19}$$

The following lemma prove the link travel time defined above satisfy the FIFO rule.

**Lemma 1**: The dynamic link travel time function of the travelers of mode $m$ meet the FIFO condition. On the other hand, a user of a certain modes entering a link later should not leave that link earlier than another user of that same mode that already drives on the same link. The FIFO condition only needs to be satisfied within each mode, but need not be satisfied across modes.

**Proof**: The FIFO condition can be expressed mathematically as

$$k - 1 + t_{am}(k - 1) \leq k + t_{am}(k) \Rightarrow t_{am}(k - 1) - t_{am}(k) \leq 1$$

$$\Rightarrow q_a(k + t_{am} - 1) - q_a(k + t_{am}) \leq \delta \cdot s_a \tag{20}$$

The above inequality means the travelers of mode $m$ entering into link $a$ during time interval $k$ must leave the link later than the travelers of mode $m$ entering into link $a$ during time interval $k-1$.

The following equation is given according to equation (11,12,13,14)

$$q_a(k + t_{am}) = \max \{0, (v_a(k + t_{am}) - s_a)\delta\} \tag{21}$$

Where

$$v_a(k + t_{am}) = \sum_m pcu_m (v_{am}^*(k + t_{am}) + q_{am}(k + t_{am} - 1)/\delta)$$

$$= v_{am}^*(k + t_{am}) + q_{am}(k + t_{am} - 1)/\delta \tag{22}$$

From equation (21), we know that there are only two possible cases for the total queue experienced by the travelers of mode $m$ entering into link $a$ during time interval $k$. One case is $q_a(k + t_{am}) = 0$, other is $q_a(k + t_{am}) = (v_a(k + t_{am}) - s_a)\delta$. When $q_a(k + t_{am}) = 0$, we can get $v_{am}^*(k + t_{am}) \leq s_a$ according to equation (21). Since $v_{am}^*(k + t_{am})$ is nonnegative, so, $q_a(k + t_{am} - 1)/\delta \leq s_a$, which leads to the inequality (20).

Another case $q_a(k + t_{am}) = (v_a(k + t_{am}) - s_a)\delta = v_{am}^*(k + t_{am}) \cdot \delta + q_{am}(k + t_{am} - 1) - s_a \delta$

leads to the inequality(20) directly because $q_{am}(k + t_{am} - 1) - q_{am}(k + t_{am}) \geq s_a \delta$ under the nonnegative assumption for all flow rates of the travelers of mode $m$.

The proof completes.
2.4 Link And Path General Travel Cost

The travelers are assumed in making travel choices to only consider the travel time cost or the schedule delay cost. However, there are many factors to affect the travel choice behaviors of travelers such as other travel cost etc. these may include safety, comfort fuel consumption and environmental factors etc. Here, we consider the amount of fuel consumption is directly relative with the economic benefit of the travelers. Thus, we assume the use of multiple criteria in decision-making as regards travels choices, and, in particular, route and departure time selection, allows travelers to not only capture travel time as a criteria but also cost of fuel consumption as another criteria. On the other hand, travelers are multi-criteria decision-maker in that they perceive their disutility associated with selecting routes and departure times as a weighting of travel time, cost of fuel consumption and schedule delay.

We are now ready to describe the general travel cost function associated with the links. Here, the link travel time cost function of mode \( m \) associated with each link \( a \) in the network is given as

\[
c_{am}(k) = \alpha \cdot t_{am}(u_a), \forall a, m
\]  

(23)

Where \( u_a = \{\hat{u}_{am}(k), \bar{u}_{am}(k)\}, \forall k, m \) express all inflow rate set of equipped and unequipped travelers of mode \( m \) on link \( a \) for all time interval. \( \alpha \) is a convention factor (the cost of one unit of travel time, RMB/hour) to transform travel time \( t_{am}(u_a) \) into travel cost \( c_{am}(k) \).

There are a number of fuel consumption models of various complexities. for the analyses performed in this paper a macroscopic relationship used in TRANSYT-7F model given by Penic,M.A. (1993). Here, in order to use easily in dynamic network model framework, the extended function form of the model is given as

\[
ROP_{am}(k) = \frac{A \exp(B \cdot \bar{v}_{am}(k))}{\bar{v}_{am}(k)}, \forall a, m
\]  

(24)

Where

\( ROP_{am}(k) \) is the rate of production (fuel consumption: gal-vehicle/km) of the travelers of mode \( m \) entering into link \( a \) during time interval \( k \)

\( \bar{v}_{am}(k) \) is the average velocity(km/h) of the travelers of mode \( m \) entering into link \( a \) during time interval \( k \)

\( A, B \) is the parameters.

The average velocity (km/hour) of the travelers of mode \( m \) entering into link \( a \) during time interval \( k \) is derived by dividing the distance of the link by the travel time, \( \bar{v}_{am}(k) = l_a / t_{am}(k) \). \( l_a \) is the distance of the link \( a \). the cost of fuel consumption produced by per traveler of mode \( m \) entering into link \( a \) during time interval \( k \) is calculated by multiplying the production rate by the distance of the link and the relative parameters.
\( f_{am}(k) = \beta \cdot ROP_{am}(k) \cdot t_a \) \hspace{1cm} (25)

\( \beta \) is a convention factor (the cost of one unit of fuel consumption, RMB/gal) to transform the amount of fuel consumption into travel cost.

We assume that each mode of travelers has his own perception of the trade-off between travel time cost and fuel consumption cost which are represented by the nonnegative weights \( w^1_{am} \) and \( w^2_{am} \). Here \( w^1_{am} \) express the weight associated with the mode \( m \)'s travel time cost, \( w^2_{am} \) express the weight associated with the mode \( m \)'s fuel consumption cost. The weights \( w^1_{am} \) and \( w^2_{am} \) are link-dependent and, hence, can incorporate such link-dependent factors as safety, comfort, and view. For example, in the case of a pleasant view on a link, travelers may weight the fuel consumption cost higher than the travel time on such a link. However, if a link has a rough surface or is noted for unsafe road conditions such as ice in the winter, travelers may then assign a higher weight to the travel time cost than the fuel consumption cost, because travelers can choose low velocity for safety. Link-dependent weights provide a greater level of generality and flexibility in modeling travel decision-making than weights that are identical for the travel time cost and for the fuel consumption cost on all links for a given mode (see Nagurney, 2000 2001).

We construct the link general travel cost function of mode \( m \) with associated with link \( a \), and denote by \( U_{am}(k) \), as

\[ U_{am}(k) = w^1_{am} \cdot c_{am}(k) + w^2_{am} \cdot f_{am}(k), \forall a, m, k \] \hspace{1cm} (26)

We may write

\[ U_{am}(k) = U_{am}(u_a), \forall a, m, k \] \hspace{1cm} (27)

Now we construct the path travel time function and the general path travel cost function by the nested function.

The path travel time function is given as

\[ t_{pm}^{rs}(k) = t_{a,m}(k) + t_{a,m}(k + t_{a,m}(k)) + \cdots + t_{a,m}(k + t_{a,m} + t_{a,m} + \cdots + t_{a,m}), \forall rs, p, m \] \hspace{1cm} (28)

for short \( t_{a,m} = t_{a,m}(k), t_{a,m} = t_{a,m}(k + t_{a,m}(k)) \).

The general path travel cost function is given as

\[ U_{pm}^{rs}(k) = U_{a,m}(k) + U_{a,m}(k + t_{a,m}(k)) + \cdots + U_{a,m}(k + t_{a,m} + t_{a,m} + \cdots + t_{a,m}), \forall rs, p, m \] \hspace{1cm} (29)

The path travel time function and the general path travel cost function can be rewritten as

\[ t_{pm}^{rs}(k) = \sum_{a \in p} \sum_{l : k \in K} t_{am}(l) \delta_{amk}(l) \] \hspace{1cm} (30)

\[ U_{pm}^{rs}(k) = \sum_{a \in p} \sum_{l : k \in K} U_{am}(l) \delta_{amk}(l) \] \hspace{1cm} (31)
Where $\delta_{rpmk}^r(l)$ is equal to 1, if the flow of the travelers of mode $m$ on path $p$ for OD pair $(r, s)$ entering the network at interval $k$ arrives link $a$ at interval $l$; otherwise, 0. This is written as

And for any link $a$ on path $p$

$$\sum_{l(\in l)\in K} \delta_{rpmk}^r(l) = 1$$

(32)

This is written as $\delta_{rpmk}^r(l) = \begin{cases} 1, & \text{if } k + t_{a,m} + t_{a,m} + \cdots + t_{a,m} = l \\ 0, & \text{otherwise,} \end{cases}$

Consider the schedule delay cost function as follow.

$$Sch_s(k) = \begin{cases} \beta [t_s - \Delta_s - k\delta] & \text{if } t_s - \Delta_s > k\delta \\ \gamma [k\delta - t_s - \Delta_s] & \text{if } t_s + \Delta_s < k\delta \\ 0 & \text{otherwise} \end{cases} \forall k$$

(33)

Denote $[t_s - \Delta_s, t_s + \Delta_s]$ as the desired time interval of the travelers for arrival at the destination $s$ in the network. Where $t_s - \Delta_s$ is the commuter’s desired earliest arrival time of the travelers at the destination $s$, $t_s + \Delta_s$ is the desired latest arrival time of the travelers at the destination $s$. $\beta$, $\gamma$ is the unit cost of schedule delay early, late of the travelers at the destination $s$.

Therefore, the generalized travel cost of a trip from origin $r$ to destination $s$ on path $p$ for the travelers of mode $m$ leaving origin $r$ during time interval $k$ is

$$c_{rpm}^r(k) = U_{pm}^r(k) + Sch_s(k)$$

(34)

### 3 Dynamic User Equilibrium Conditions

In most of previous multi-class dynamic network model (Bin.R 1996, Hong.K 1996, Williams and Huang 2003 etc) the equipped travelers who may receive the real-time perfect traffic information is assumed to make travel choice in a deterministic dynamic user equilibrium manner. In reality, traffic information is rarely perfect. There are many difficulties for estimating the current traffic information and predicting the future traffic information in current traffic technology condition. A greater number of the equipped travelers may select the best alternatives (from their individual point of view) and consequently the equipped travelers with similar preferences will tend to concentrate on the same routes during the same departure times. Thus, more perfect information could potentially generate higher levels of traffic congestion. The traffic manager and individual travelers don’t hope this condition happen. For these reasons, we model the travel choice behavior of the equipped travelers as following the
principle of stochastic dynamic user equilibrium. The traffic information quality is reflected by the parameter of general travel cost perception variation. Better information qualities have lower travel cost variation. The unequipped travelers in making travel choices, in particular, route and departure time choice, according to the past experiences that don’t capture better information than the equipped travelers follow another stochastic dynamic user equilibrium with higher travel cost variation.

Vythoulkas (1990), Williams and Huang (2003) adopted the nested logit model with the two parameter to express the uncertainty of the route and departure time choices. For simplicity, we assume two parameter is same.

\( \hat{f}_{pm}(k), \tilde{f}_{pm}(k) \) the inflow rate of equipped and unequipped travelers of mode \( m \) entering the path \( p \) between origin \( r \) and destination \( s \) during time interval \( k \).

\( \hat{f}, \tilde{f} \) the set of \( \{\hat{f}_{pm}(k), \forall rs, p, k, m\}, \{\tilde{f}_{pm}(k), \forall rs, p, k, m\} \).

\( \hat{q}_{pm}, \tilde{q}_{pm} \) the demand of equipped and unequipped travelers of mode \( m \) between origin \( r \) and destination \( s \).

\( \hat{P}_{pm}^{rs}(k), \tilde{P}_{pm}^{rs}(k) \) the proportion of equipped and unequipped travelers of mode \( m \) between origin \( r \) and destination \( s \) selecting path \( p \) and departure time \( k \).

\( \hat{\theta}_m \) the parameter representing travel cost variation of equipped travelers of mode \( m \).

\( \tilde{\theta}_m \) the parameter representing travel cost variation of unequipped travelers of mode \( m \).

The equipped travelers of mode \( m \) follow the stochastic dynamic simultaneous route and departure time equilibrium (SUE-SRD), expressed as

\[
\hat{f}_{pm}(k) = \hat{P}_{pm}^{rs}(k) \cdot \hat{q}_{pm}^{rs} \quad \forall rs, p, k, m
\]  

(35)

Where

\[
\hat{P}_{pm}^{rs}(k) = \frac{\exp(-\hat{\theta}_m \cdot c_{pm}^{rs}(k))}{\sum_p \sum_k \exp(-\hat{\theta}_m \cdot c_{pm}^{rs}(k))} \quad \forall rs, p, k, m
\]  

(36)

\( \hat{\theta}_m \) is the parameter representing general travel cost perception variation of the equipped of mode \( m \). A higher \( \hat{\theta}_m \) means smaller general travel cost variation and better information quality.

The logit-based SUE-SRD of the equipped travelers of mode \( m \) can be expressed as
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\[ \hat{C}_{pm}^\infty(k, f^*) = \begin{cases} \hat{c}_{m, \text{min}}^\infty & \text{if } \hat{r}^\infty_{pm}(k) > 0 \\ > \hat{c}_{m, \text{min}}^\infty & \text{otherwise} \end{cases} \quad \forall rs, k, p, m \tag{37} \]

\[ \sum_p \sum_k \hat{r}^\infty_{pm}(k) = \frac{d_m^\infty}{\delta} \quad \forall rs, m \tag{38} \]

\[ \hat{C}_{pm}^\infty(k, f^*) = c_{pm}^\infty(k) + \frac{1}{\theta_m^\infty} \ln \hat{f}^\infty_{pm}(k) \quad \forall rs, k, p, m \tag{39} \]

\[ \hat{f}^\infty_{pm}(k) \geq 0 \quad \forall rs, k, p, m \tag{40} \]

Where, \( \hat{c}_{m, \text{min}}^\infty \) is the minimum perceived unit travel cost of the equipped travelers of mode \( m \) between origin \( r \) and destination \( s \), \( \hat{c}_{m, \text{min}}^\infty = \min \hat{c}_{pm}^\infty(k, f^*), \forall p, k \). \( \hat{C}_{pm}^\infty(k, f^*) \) is perceived unit travel cost incurred by the equipped travelers of mode \( m \) entering path \( p \) between origin \( r \) and destination \( s \) during time interval \( k \). equation(38) represent the flow conservation of the equipped travelers of mode \( m \) between origin \( r \) and destination \( s \) and equation(40) represent the non-negative of all path inflow rate. For each mode of equipped travelers and for each origin-destination (OD) pair, the perceived path travel costs experienced for all equipped travelers of mode \( m \), regarding of the departure time, is equal and minimum, and less than (or equal to) the perceived path travel costs for the equipped travelers of mode \( m \) on any unused route.

The above SUE-SRD equilibrium condition of the equipped travelers of mode \( m \) can be expressed by a finite dimensional variational inequality formulation.

Find a vector \( \hat{f}^* \in \hat{\Omega} \) if and only if it satisfy

\[ \sum_{rs} \sum_p \sum_k \sum_m \hat{C}_{pm}^\infty(k, f^*)(\hat{f}_{pm}^\infty(k) - \hat{f}^\infty_{pm}(k)) \geq 0 \quad \forall \hat{f}^* \in \hat{\Omega} \tag{41} \]

where , \( \hat{\Omega} \) is a closed convex.

\[ \hat{\Omega} = \left\{ \hat{f} \left| \sum_p \sum_k \hat{r}^\infty_{pm}(k) = \frac{d_m^\infty}{\delta}, \hat{f}_{pm}^\infty(k) \geq 0, \forall rs, m \right. \right\} \tag{42} \]

VI formation (41) can be expressed as a standard form

\[ \left\langle \hat{C}(\hat{f}^*), \hat{f} - \hat{f}^* \right\rangle \geq 0 \quad \forall \hat{f} \in \hat{\Omega} \tag{43} \]

Where \( \left\langle \cdot, \cdot \right\rangle \) denoted the inner product of Euclidean space

The treatment of the unequipped travelers of mode \( m \) is identical. Without loss of generality, one may write:

where
\[
\tilde{P}_{pm}(k) = \frac{\exp(-\tilde{\theta}_m \cdot c_{pm}(k))}{\sum_p \sum_k \exp(-\tilde{\theta}_m \cdot c_{pm}(k))} \quad \forall rs, p, k, m
\]  
\[
\tilde{\theta}_m \text{ express the travel cost perception variation of the unequipped travelers of mode } m \text{ that can be interpreted as their familiarity of the network condition or the past experiences.}
\]

The logit-based SUE-SRD of the unequipped travelers of mode \( m \) can be expressed as
\[
\tilde{C}^{rs*}_{pm}(k, f^*) = \begin{cases} 
\tilde{c}^{rs}_{pm,\text{min}} & \text{if } \tilde{f}^{rs}_{pm}(k) > 0 \\
\tilde{c}^{rs}_{pm,\text{min}} & \text{otherwise} 
\end{cases} \quad \forall rs, k, p, m
\]  
\[
\sum_p \sum_k \tilde{f}^{rs}_{pm}(k) = \frac{\tilde{q}_{m}}{\delta} \quad \forall rs, m
\]  
\[
\tilde{C}^{rs*}_{pm}(k, f^*) = c^{rs}_{pm}(k) + \frac{1}{\tilde{\theta}_m} \ln \tilde{f}^{rs*}_{pm}(k) \quad \forall rs, k, p, m
\]  
\[
\tilde{f}^{rs}_{pm}(k) \geq 0 \quad \forall rs, k, p, m
\]

The above SUE-SRD equilibrium condition of the unequipped travelers of mode \( m \) can be expressed by a finite dimensional variational inequality formulation.

Find a vector \( \tilde{f}^* \in \Omega \) if and only if it satisfy
\[
\sum_{rs} \sum_{p} \sum_{k} \sum_{m} \tilde{C}^{rs*}_{pm}(k, f^*)(\tilde{f}^{rs}_{pm}(k) - \tilde{f}^{rs*}_{pm}(k)) \geq 0 \quad \forall \tilde{f} \in \tilde{\Omega}
\]  
where , \( \tilde{\Omega} \) is a closed convex.

\[
\tilde{\Omega} = \left\{ \tilde{f} \bigg| \sum_p \sum_k \tilde{f}^{rs*}_{pm}(k) = \frac{\tilde{q}_{m}}{\delta}, \tilde{f}^{rs}_{pm}(k) \geq 0, \forall rs, m \right\}
\]

VI formation (51) can be expressed as a standard form
\[
\langle \tilde{C}(\tilde{f}^*), \tilde{f} - \tilde{f}^* \rangle \geq 0 \quad \forall \tilde{f} \in \tilde{\Omega}
\]
3.1 The Composite VI Formulation

The composite VI problem that integrate the VI (41) with VI (51) is equivalent to the above user equilibrium condition (37) and (47)

The composite VI model can be formulated as follows:

Find a vector \((\hat{\Omega}, \check{\Omega})\) that is a multi-mode, multi-class stochastic dynamic user equilibrium pattern if and only if it satisfies the VI problem

\[
\sum_{rs} \sum_{p} \sum_{k} \sum_{m} \hat{C}_{pm}^{rs}(k, \hat{f}^{rs}_{pm}(k) - \hat{f}_{pm}^{rs}(k)) + \sum_{rs} \sum_{p} \sum_{k} \sum_{m} \check{C}_{pm}^{rs}(k, \check{f}^{rs}(k) - \check{f}^{rs}_{pm}(k)) \geq 0
\]

\[\forall \hat{f} \in \hat{\Omega}, \forall \check{f} \in \check{\Omega}\] (54)

Where \(\hat{\Omega}\) and \(\check{\Omega}\) are the sets of all feasible path inflow rate with all departure time associated with equipped and unequipped travelers of mode \(m\).

The composite VI (54) can be expressed as a standard form

\[
\langle \hat{C}, \hat{f} - \hat{f}^{*} \rangle + \langle \check{C}, \check{f} - \check{f}^{*} \rangle \geq 0 \quad \forall \hat{f} \in \hat{\Omega} \quad \forall \check{f} \in \check{\Omega}
\] (55)

4. Algorithm

The optimal solution to variational inequality (54) can be found in the framework of the diagonalisation method (Ran and Boyce, 1996). Before describing the diagonalisation method in detail, we will see how we can perform stochastic dynamic network loading, which is essential to feasible link flow patterns. Note that this paper particularly develops a stochastic dynamic network loading method considering the logit-based route and departure time choice. The logit-based route and departure time choice model can be written as follows:

\[
P_{rs}^{k}(k) = \frac{\exp(-\theta \cdot c_{p}^{rs}(k))}{\sum_{p} \sum_{k} \exp(-\theta \cdot c_{p}^{rs}(k))} \quad \forall rs, p, k
\] (56)

4.1 Dynamic Stochastic Network Loading Method

In this section, stochastic dynamic network loading algorithm for the logit-based route and departure time choice is proposed. This network loading algorithm is similar to the algorithm proposed by Dial’s STOCH for stochastic static network assignment and the algorithm proposed by B.Ran’s DYNASTOCH for stochastic dynamic network assignment. In this study, we consider only the logit model for stochastic dynamic simultaneous route/departure time choice. The algorithm maintains the structure of the DYNASTOCH algorithm, so only deals with reasonable routes, and assigns the demand between OD pair \(rs\) to the link of the network according to the actual link travel cost.(denote the stochastic dynamic network loading method as SRD-DYNASTOCH)
In order to reflect the effect of the schedule delay cost in the algorithm, we extend the original network to include the dummy link with schedule delay cost, $c^{is}(k) = sch_i(k), \forall i$ as shown in Fig.2.

![Fig.2: the extended network structure](image)

Step 1: Calculation of link likelihood

Compute the minimum actual travel cost $\pi_{is}(k)$ for travelers departing node $i$ during time interval $k$. Calculate the link likelihood, $L_{(i,j)}(k)$, for each link $(i,j)$ during each time interval $k$:

$$L_{(i,j)}(k) = \begin{cases} 
\exp(\theta[\pi_{is}(k) - \pi_{js}(k + t_{(i,j)}(k)) - c_{(i,j)}(k)]) & \text{if } C^{is}_o > C^{js}_o \\
0 & \text{otherwise} \end{cases}$$

$$i \in r$$

$$L_{(i,j)}(k) = \begin{cases} 
\exp(\theta[\pi_{is}(k) - \pi_{js}(k + t_{(i,j)}(k)) - c_{(i,j)}(k)]) & \text{if } C^{is}_o > C^{js}_o \\
0 & \text{otherwise} \end{cases}$$

$$i \notin r$$

$\pi_{is}(k)$ is the minimum travel cost from $i$ to $s$ by departing the node $i$ during time interval $k$

$\pi_{rs}$ is the minimum path travel cost from origin $r$ to destination $s$ for all departure time.

$\pi_{rs} = \min\{c^{rs}_p(k), \forall p, k\} \ \forall rs$

$C^{is}_o$ is the ideal travel cost from $i$ to $s$ when there is no flow in the network

$t_{(i,j)}(k)$ is link travel time experienced by the travelers entering into link $(i,j)$ during time interval $k$

$c_{(i,j)}(k)$ is link travel cost experienced by the travelers entering into link $(i,j)$ during time interval $k$

Step 2: backward pass

By examining all nodes $j$ in ascending sequence with respect to $\pi_{is}(k)$ from the destination $s$, calculate $w_{(i,j)}(k)$, the link weight for each link $(i,j)$ during each time interval $k$: 

#.16
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\[ w_{(i,j)}(k) = \begin{cases} 
L_{(i,j)}(k) & \text{if } j = s \\
L_{(i,j)}(k) \cdot \sum_{(j,k) \in A(j)} w_{(j,k)}(k + t_{(j,k)}(k)) & \text{otherwise}
\end{cases} \tag{59} \]

Where \( A(j) \) is the set of links starting from node \( j \). When the origin \( r \) is reached, stop.

Step 3: forward pass

Consider all nodes \( i \) in descending sequence with respect to \( \pi_{ik}(k) \), starting with the origin \( r \). When each node \( i \) is considered during each time interval \( k \), compute the inflow to each link \( (i,j) \) during each time interval \( k \) using the following formula:

\[ v_{(i,j)}(k) = \begin{cases} 
q_{ni} \cdot \frac{w_{(i,j)}(k)}{\sum_{k} \sum_{(i,k) \in A(i)} w_{(i,k)}(k)} & \text{if } i = r \\
\left( \sum_{(k,j) \in B(i)} u_{(k,j)}(k) \right) \cdot \frac{w_{(i,j)}(k)}{\sum_{(i,k) \in A(i)} w_{(i,k)}(k)} & \text{otherwise}
\end{cases} \tag{60} \]

Where, \( B(i) \) is the set of links ending at node \( i \). When the destination \( s \) is reached, stop.

The flow generated by the algorithm is equivalent to a logit-based flow independent route/departure time assignment between each OD pair, given the reasonable route set is fixing in order to produce a convergence solution.

The proofs of the algorithm see Appendix.

4.2 Diagonalisation Algorithm

Here, we propose the diagonalisation method to solve multi-mode, multi-class and multi-criteria stochastic dynamic simultaneous route/departure time equilibrium problem. The method is similar with the algorithm of B.Ran(1996), H.K. Chen(1998) and Han (2003). the algorithm consist of the outer iteration and inner iteration, outer iteration includes the updating estimation of actual link travel time or link inflow rate. Inner iteration calculates the link inflow updating direction and the auxiliary link inflow rate by the method of successive averages. The processes of algorithm are stated as follows.

Step 0: Initialization: Set outer iteration counter \( i = 1 \), perform \( 2 \times m \) stochastic dynamic network loading (SRD-DYNASTOCH) for the given demand \( \hat{q}^{rs}_{m}, \tilde{q}^{rs}_{m} \) according to free flow travel cost, find initial link inflow rate \( \hat{u}_{im}^{i}(k), \tilde{u}_{im}^{i}(k) \)
Step 1: inner iteration (MSA)

Step 1.0 initialization: set inner iteration counter \( j = 1 \), \( \hat{u}^i_{am}(k) = \tilde{u}^i_{am}(k) = \widetilde{u}^i_{am}(k) \)

Step 1.1 calculate link travel time and travel cost \( t_{am}^i(k), c_{am}^i(k) \) by using

\[ \hat{u}^i_{am}(k), \tilde{u}^i_{am}(k). \]

Step 1.2 direction finding: perform \( 2m \) stochastic dynamic network loading (SRD-DYNASTOCH) for the given demand \( \tilde{q}_m, \tilde{q}_m \), according to current actual link travel time and travel cost \( t_{am}^i(k), c_{am}^i(k) \). This generates auxiliary link flow \( \hat{u}^i_{am}(k), \tilde{u}^i_{am}(k) \)

Step 1.3 move: update flow pattern as

\[ \hat{u}^{i+1}_{am}(k) = \hat{u}^i_{am}(k) + \lambda^i (\hat{u}^i_{am}(k) - \tilde{u}^i_{am}(k)) \]
\[ \tilde{u}^{i+1}_{am}(k) = \tilde{u}^i_{am}(k) + \lambda^i (\tilde{u}^i_{am}(k) - \hat{u}^i_{am}(k)) \]

Step 1.4 (convergence test of inner iteration) if

\[
\frac{\sum_m \sum_a (\hat{u}^{i+1}_{am}(k) - \hat{u}^i_{am}(k))^2 + (\tilde{u}^{i+1}_{am}(k) - \tilde{u}^i_{am}(k))^2}{\sum_m \sum_{a} (\hat{u}^i_{am}(k) + \tilde{u}^i_{am}(k))} \leq \gamma \quad (\gamma \text{ is a predetermined tolerance})
\]

or \( j \) is equal to a given number, then stop; otherwise, go to step 1.1 and set \( j = j + 1 \).

Step 2 convergence test of outer iteration: if the convergence criteria are satisfied or \( i \) is equal to a given number, stop; otherwise, go to step 1 and set \( i = i + 1 \). The step size \( \lambda^i \) is a predetermined value, we set \( \lambda^i = 1/j \), or \( \lambda^i = 1 \). In order to maintain correct flow propagation constraint, we calculate new link flow pattern by directly updating link choice probability (Han(2003)) or use pure network loading.

5 Examples

In this section, the above algorithm finds the solution of multimode, multi-class and multi-criteria stochastic dynamic simultaneous route/departure time equilibrium model and we perform only one iteration in the inner iteration of the diagonalization method according to Sheffi (1985)’s advice. For stochastic dynamic simultaneous route/departure time network loading, we use the SRD-DYNASTOCH algorithm in order to perform a logit-based flow assignment.
Here we assume there are two modes in the network, one is car, and another is truck. The example network, shown in Fig.4, consists of 3 nodes, 4 links and one OD pair. The free flow travel time of car and truck, link exit capacities and link length are also given in this figure. The passenger car equivalents parameter of car and truck is $P_{cu_1}=1, P_{cu_2}=2$. Other input data are: $\alpha=6(\text{RMB/h}), \beta=4(\text{RMB/h}), \gamma=22(\text{RMB/h}), \Delta s=0.25h, t_s=9.0h$, and $T$ be from 5 to 12 a.m. and $K=600$, $\delta=0.6min$. Here we assume the perception parameter of each mode is same. $\hat{\theta}_1=\hat{\theta}_2=0.1, \hat{\theta}_1=\hat{\theta}_2=0.05$. The parameter of fuel consumption model of car (truck) is given as $A_1=1.4, B_1=0.02, A_2=1.4, B_2=0.022$.

The parameter values show the economic velocity range of car (truck) is 50 - 60km/h (40 - 50km/h). Beyond this range, the fuel consumption of vehicles will increase, and at lower velocity, very quickly. The coefficients of link weight is: $w_{11}^1 = 0.5, w_{12}^2 = 0.25$ (link 1), $w_{21}^1 = 0.5, w_{23}^2 = 0.5, w_{22}^1 = 0.75, w_{22}^2 = 0.25$ (link 2), $w_{31}^1 = 0.5, w_{31}^2 = 0.5, w_{32}^1 = 0.75, w_{32}^2 = 0.25$ (link 3), and $w_{41}^1 = 0.5, w_{41}^2 = 0.5, w_{42}^2 = 0.5$ (link 4).

The demand of car and truck is $q_1^r(k)=15000$ (persons), $q_2^r(k)=12000$ (persons). The demand of equipped and unequipped of car (truck) is $\hat{q}_1^r(k)=\hat{q}_2^r(k)=50\% q_1^r(k)$, $\hat{q}_2^r(k)=\hat{q}_2^r(k)=50\% q_2^r(k)$. Here, it is further assumed that the vehicle occupancy is 1 person per vehicle (car and truck).
Firstly, we check the effectiveness of the proposed approach for solving the multi-mode, multi-class and multi-criteria dynamic user equilibrium problem. The situation where all parameters are valued as above is termed “base case”. We will investigate the consequence of the algorithm convergence when each time changing one parameter and remaining others with the same as in the base case. Fig.4 gives the convergence performance of the algorithm in some cases differentiating from $\theta_m, \tilde{\theta}_m (m=1,2)$ values. We can see that the algorithm convergences rapidly, particularly in the first several iterations when the value of $\theta_m, \tilde{\theta}_m$ is small. However, when $\tilde{\theta}_m = 0.5$, we can not find the convergence performance of the algorithm. In other words, the convergence solution of logit-based stochastic assignment can not be found if $\tilde{\theta}_m, \tilde{\theta}_m$ is too large (see Han, 2000).

The inflow rates of equipped (unequipped) travelers of car (truck) are shown on link 1,2,3 and 4 in Fig.5, 6,7,8.
Fig. 5. link 4's inflow rate of truck and car

Fig. 6. link 2's inflow rate of truck and car
Fig. 7. Link 3's inflow rate of truck and car

Fig. 8. Link 4's inflow rate of truck and car
We can find the equipped travelers of car (truck) are more concentrated on the limit departure time than the unequipped travelers of car (truck) due to the fairly perfect traffic information. The formation and dissipation of the queues are depicted on some links in Fig.9. There are many queues on link 1 and 2. However the queues of link 4 are few (only car) while the queue of link 3 are none since the exiting rates of their upstream links are less than or equal to their capacities.

Now we investigate the ATIS impacts on the each mode of travelers and system performance with respect to the ATIS market penetration and the information quality of equipped travelers $\hat{\theta}_m$. Total market penetration of two modes is $\eta = \frac{\hat{q}_{1s}^c + \hat{q}_{2s}^c}{q_1^{cs} + q_2^{cs}}$, the market penetration of car is $\eta_1 = \frac{\hat{q}_{1s}^c}{q_1^{cs}}$, the market penetration of truck is $\eta_2 = \frac{\hat{q}_{2s}^c}{q_2^{cs}}$. For simplicity, we assume $\eta_1 = \eta_2$. The value of general travel cost perception parameter $\hat{\theta}_m$ of the unequipped travelers of car (truck) is fixed as 0.01. The demand of car and truck is $q_1^{cs}(k) = 15000, q_2^{cs}(k) = 15000$.

![Fig.9. link queue of truck and car](image-url)
Fig. 10 and 11 depict the individual average travel costs of equipped and unequipped travelers of car (truck) against the total market penetration $\eta$ and the travel cost perception parameter of the equipped travelers of car (truck) $\hat{\theta}_m$ ($\hat{\theta}_m = 0.05, 0.1, 0.15$ as shown in the legend). It is shown that the average travel cost of equipped travelers of car (truck) is higher than the unequipped travelers of car (truck) at the different market penetration and the value of $\hat{\theta}_m$. Implying that using ATIS always benefits the equipped travelers of different mode if we neglect the cost for purchasing the ATIS device and using the information system. The average travel cost of equipped and unequipped travelers of car (truck) are ascending with the increase of market penetration and the value of $\hat{\theta}_m$. We can find the average travel cost saving of equipped travelers of car (truck) versus the unequipped travelers of car (truck) is marginally descending when the market penetration is more than 30%. It is shown that ATIS effects to the equipped and unequipped travelers of car (truck) are negative in many conditions. This could be explained as follows. With the increase of the market penetration and the value of $\hat{\theta}_m$, the equipped travelers of car (truck) that affect the ability of the travelers and traffic system are superior to the unequipped travelers of car (truck).
It can be seen from Fig. 5 and 6 that the equipped travelers of car (truck) choose the range of departure time and route narrower than the unequipped travelers of car (truck) since receiving the more traffic information. In other hands, A greater number of the equipped travelers of car (truck) may select the best alternatives (from their individual point of view) and consequently the equipped travelers of car (truck) will tend to concentrate on the same routes during the same departure times. Thus, higher levels of traffic congestion could potentially be generated by more information and higher market penetration. Finally the benefits of the equipped travelers of car (truck) could be reduced.

It can be seen from Fig. 10 and 11 that the average travel cost of the equipped travelers of car (truck) become small and the average travel cost of the unequipped travelers of car (truck) become very small with the increase of the value of $\theta_m$ when the market penetration is less than 40%. And we can find the change of the system total travel cost and total travel time is very small in Fig.12 and 13. In other words, when the market penetration is small, the equipped travelers of car (truck) receiving more perfect traffic information can get more benefits. However, when the market penetration is more than 40%, the results turn upside down. Not only the average travel cost of equipped and unequipped travelers of car (truck) ascends, but also the average travel cost saving of equipped travelers of car (truck) descends and it can be seen from Fig.12 and 13 the system total travel cost and total travel time increases rapidly.
Fig. 12 Total travel cost with respect to Market penetration

Fig. 13 Total travel time with respect to Market penetration
In other words, when the market penetration is high, the equipped travelers of car (truck) for receiving more perfect traffic information can get smaller benefits and the system traffic condition will exacerbate. The results give us an alarm that the bad results could be induced if the more perfect traffic information is provided to the equipped travelers of car (truck) in some cases.

Fig 12 and 13 depicts system total travel cost and total travel time with respect to the market penetration and the value of $\hat{\theta}_m$. It can be seen from Fig 12 and 13 that system total travel cost and total travel time increase with the increase of the market penetration and the value of $\hat{\theta}_m$, when the market penetration is more than 30%. In other hand, the ATIS is most likely to generate negative effect on the transport network.

Fig.14 gives total fuel consumption amount with respect to the market penetration and the value of $\hat{\theta}_m$. When the market penetration is small, total fuel consumption amount is decreasing. At this level of small market penetration, the network is not congestion, thus the travelers can run at higher velocity (when the velocity of vehicle is high or low, the fuel consumption will increase). The network becomes congested with the increase of market penetration and.

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Fig.14 Total fuel consumption with respect to Market penetration
```

the velocity of vehicle is induced to lower to the range of economic velocity, thus total fuel consumption amount is ascending. When market penetration is more than a value, with the increase of market penetration and the value of $\hat{\theta}_m$, the network become very congested, the travelers running at low velocity will lead to the increase of the fuel consumption.

6 Conclusions

This paper presents a formulation and solution algorithm for multi-mode, multi-class and multi-criteria dynamic user equilibrium problem, in order to assess the effects of ATIS in recurrent congestion network with queues. Suppose the each mode of equipped travelers that receive the traffic information follow stochastic dynamic simultaneous route and departure time user-equilibrium with small travel cost perception variation, the each mode of unequipped travelers in making travel choices according to the past experiences follow stochastic dynamic simultaneous route and departure time user-equilibrium with high travel cost perception variation. We consider the each mode of the equipped (unequipped) travelers are multi-criteria decision-maker in that they perceive their disutility associated with selecting routes as a weighting of the travel times and travel costs. A diagonalization algorithm based on stochastic dynamic network loading for logit-based simultaneous route and departure time is proposed. Finally a numerical example is presented to demonstrate the ATIS impacts on each mode of individual average travel cost, total travel cost, total travel time and total fuel consumption etc at the different market penetration and the value of the equipped traveler’s travel cost perception variation in a simple network.

In further studies. 1. The calibration of the model parameter such as travel cost perception variation of the equipped and unequipped travelers, value of time and weight coefficients etc. 2. Considering multi-mode dynamic network model with physical queue. 3. Model in application to assessing the impacts of the environment pollution etc.

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**Appendix:** Proof of the Algorithm
We now prove that the algorithm does generate logit-based flow independent ideal stochastic dynamic simultaneous route and departure time choices between each OD pair. We note that each link likelihood \( L_{(i,j)}(k) \) is proportional to the logit probability that link \( a=(i,j) \) is used during time interval \( k \) by a traveler chosen at random from among the population of trip-makers between origin \( r \) and destination \( s \), given that the traveler is at node \( i \) during time interval \( k \). The probability that a given route will be used is proportional to the product of all the likelihoods of the links comprising this route. Suppose route \( p \) consists of nodes \( (r, 1, 2, \ldots, n, s) \) and links \( (1, 2, \ldots, h) \). Subpath \( p_1 \) includes \( (1, n, s) \) and links \( (2, \ldots, h) \). The probability of traveler choosing route \( p \) and departure time \( k \) between origin \( r \) and destination \( s \) is \( P_{rs}^p(k) \).

\[
P_{rs}^p(k) = G \cdot \prod_{a \in p} L_{(i,j)}(k)^{\delta_{ap}^r}
\]

Where \( G \) is proportionality constant for each OD pair and the product is taken over all links in the networks. Here, \( t = k + t_\alpha^p(k) \). The incidence variable \( \delta_{ap}^r \) ensures that \( P_{rs}^p(k) \) will include only those links in the \( p \)th route between origin \( r \) and destination \( s \). Substituting the expression for the likelihood \( L_{(i,j)}(k) \) in the above equation, the choice probability of choosing a particular efficient route-departure time pair becomes

\[
P_{rs}^p(k) = G \cdot \exp\left\{ \theta \left[ \pi_{rs} - \pi_{js} (k + t_{i,j}(k)) - c_{(r,j)}(k) \right] \right\} \\
\cdot \prod_{a \in p} \exp\left\{ \theta \left[ \pi_{ai} (t) - \pi_{ji} (k + t_{i,j}(k)) - c_{(i,j)}(k) \right] \delta_{ap}^r \right\}
\]

\[
= G \cdot \exp\left\{ \theta \left[ \pi_{rs} - \pi_{js} (k + t_{i,j}(k)) - c_{(r,j)}(k) \right] \right\} \\
\cdot \exp\left\{ \theta \cdot \sum_{a \in p} \left[ \pi_{ai} (t) - \pi_{ji} (k + t_{i,j}(k)) - c_{(i,j)}(k) \right] \delta_{ap}^r \right\}
\]

\[
= G \cdot \exp\left\{ \theta \cdot (\pi_{rs} - c_p^\alpha (k)) \right\}
\]

The last equality results from the following summations:

\[
\pi_{rs} - \pi_{ts} (k + t_{(r,s)}(k)) + \sum_{a \in p} \left[ \pi_{ai} (k) - \pi_{ji} (k + t_{i,j}(k)) \right] \cdot \delta_{ap}^r
\]

\[
= \pi_{rs} - \pi_{ts} (k + t_{(r,s)}^r(k)) + \pi_{1s} (k + t_{(1,s)}^r(k)) - \pi_{2s} (k + t_{(1,s)}^r(k)) + \ldots + \pi_{ns} (k + t_{(n,s)}^m(k)) - \pi_{ss} (k + t_{s}^m(k))
\]

\[
= \pi_{rs}
\]

and

\[
\sum_{a \in p} \delta_{ap}^r = c_{1}(k) + c_{2} (k + t_{i,j}^r(k)) + \cdots + c_{h}(k + t_{p}^m(k)) = c_{p}^\alpha (k)
\]

Since \( \sum_P \sum_k P_{rs}^p(k) = 1 \)
The proportionality constant must equal
\[
G = \sum \sum \frac{1}{\exp(\alpha \cdot \theta \cdot [\pi_{rs} - c_{p}^{rs}(k)])}
\]

Thus
\[
P_{rs}(k) = \frac{\exp(\theta \cdot [\pi_{rs} - c_{p}^{rs}(k)])}{\sum \sum \exp(\theta \cdot [\pi_{rs} - c_{p}^{rs}(k)])} = \frac{\exp(\theta \cdot c_{p}^{rs}(k))}{\sum \exp(\theta \cdot c_{p}^{rs}(k))}
\]

Above equation depicts a stochastic dynamic simultaneous route/departure time choice among the efficient routes connecting OD pair \(rs\). The algorithm does generate a stochastic dynamic simultaneous route/departure time choice probability using actual route travel costs.

Now we prove the forward pass of the algorithm does generate the results of the logit flow assignment for simultaneous route/departure time choice. Firstly, we transform equation (49) as the following equation.
\[
f_{p}^{rs}(k) = q^{rs} \cdot \exp(\theta \cdot c_{p}^{rs}(k)) = q^{rs} \cdot \frac{\sum \exp(\theta \cdot c_{p}^{rs}(k))}{\sum \sum \exp(\theta \cdot c_{p}^{rs}(k))}
\]

Where
\[
q^{rs}(k) = q^{rs} \cdot \frac{\sum \exp(\theta \cdot c_{p}^{rs}(k))}{\sum \sum \exp(\theta \cdot c_{p}^{rs}(k))} = q^{rs} \cdot \frac{\sum w_{i,k}(k)}{\sum w_{i,k}(k)} \quad i \in r, \forall rs, k
\]

The demand between OD pair \(rs\) during time interval \(k\) in the equation () assign to the network according to the DYNASTOCH algorithm. The equation (51) is substituted into the forward pass of the DYNASTOCH algorithm; the equation (43) can be achieved. The main difference (one is \(\pi^{rs}(k)\), other is \(\pi^{rs}\)) between the origin link ‘s the link likelihood of the DYNASTOCH algorithm and this algorithm don’t effect the result of the calculation.